MATHEMATICAL MODELING OF AVIAN INFLUENZA WITH VACCINATION AND TREATMENT FOR POULTRY FARMS

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Abstract. Avian influenza poses a significant threat to poultry farms, leading to substantial economic losses and public health concerns. Effective control strategies, including vaccination and treatment, are essential to mitigate the spread of the disease. This study aims to develop a mathematical model to understand the dynamics of avian influenza in poultry farms and evaluate the impact of vaccination and treatment interventions. A compartmental model was constructed to represent the transmission dynamics of avian influenza among poultry populations. The model includes compartments for susceptible, exposed, infected, and recovered birds, with additional compartments for vaccinated and treated birds. Differential equations were used to describe the transitions between compartments. Parameters were estimated from existing literature and field data. The model was analyzed using numerical simulations to assess the effectiveness of different intervention strategies. The model simulations indicated that a combination of vaccination and treatment significantly reduces the prevalence of avian influenza in poultry farms. Vaccination alone was effective in lowering the infection rate, but the addition of treatment further decreased the number of infected birds. Sensitivity analysis revealed that the timing and coverage of vaccination are critical factors in controlling the outbreak. Early and widespread vaccination, coupled with prompt treatment of infected birds, was found to be the most effective strategy. The mathematical model provides valuable insights into the dynamics of avian influenza and the impact of control measures in poultry farms. The findings suggest that integrated strategies involving both vaccination and treatment are essential for effective disease management. Policymakers and farm managers should consider these strategies to enhance the resilience of poultry farms against avian influenza outbreaks.

Keywords: Avian influenza, mathematical modeling, vaccination, treatment, poultry farms, disease control, epidemiology

1. Introduction

Perdue [1], Sedyaningsih [2], and Scoones & Forster [3] suggested that bird flu virus can be transmitted to humans and can cause death so that an outbreak occurs. In Yang et al [4] published in 2009, bird flu epidemic then erupted swine flu epidemic. The flu epidemic led to several deaths and many people admitted to the hospital. Strain (derived) H5N1 virus preserved as the cause of the epidemic of bird flu virus H1N1 virus as a cause of swine flu epidemic. Symptoms caused by seasonal flu are caused by H3N2 strain of the virus. Jansen et al [5] mentioned influenza viruses responsible for the number of deaths and people who are sick in the hospital. In Widiasih [6] the sense of presence in poultry infected with this virus with a very large number of economic terms very.

de Jong et al [7] mentioned that the influenza A subtype H5N1 virus with substitution of an amino acid in neuramiside isolated from 2 patients undergoing therapy/treatment and known the virus is immune to a given drug. Both of these patients died because of this viral infection. Gooskens [8] also mentioned that there is a mutation of influenza A virus that produces a virus that is immune to oseltamivir. The mutated virus is contagious pathogenic and lethal for high-risk patients. The ability of the H5N1 virus to mutate is so high that it is necessary to watch out for the spread of this virus in the poultry population so that some precautions have been taken such as the destruction of infected poultry and quarantine for infected humans.

2. Methods

The first step to do this research was literacy study. In this step, we study the fact and some assumptions from various scientific literacies. After that, we complete the facts with some assumptions to build the model. The second step was building and analyzing mathematics model. In this step, we build the mathematics model and then analyze it to determine the equilibrium points and their stability. The third step was making simulation with parameters value which was gotten from other paper.

3. Mathematical Model

From literature review we got: in Tuncer & Martcheva [9], it were stated that the avian influenzavirus of subtype H5N1 can infect humans and cause death in human and bird population. In Tuncer & Martcheva [9] and Bourouiba [10], there was stated that vaccination in poultry are still being implemented. In Bourouiba [10], it was stated that vaccinated poultry which is even free of clinical signs, should not be traded to avoid all risk of silent shedding and transmission. Vemula et al [11] used several different approaches that are currently available for diagnosis of influenza infections in humans. These are used to diagnosis of influenza virus infections following natural infection and vaccination in humans.

In this paper, we assume that the population is constant so the death rate of infected human and infected bird were assumed have same value with natural death rate in every population. We also assumed that death in infected humans and infected birds only occurs due to viral infection and the probability of infectious contacts of bird-human and human-human are same. Transfer diagram of AI epidemic is given at Figure 1.

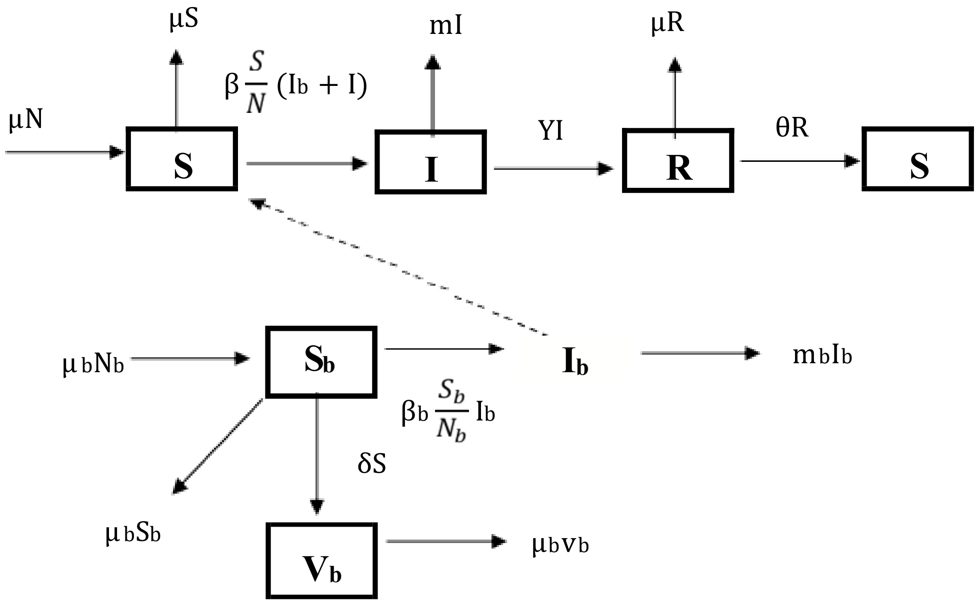


Figure 1. Transfer diagram of AI epidemic with vaccination on susceptible bird

Table 1. The meaning of parameters

Parameter	The Meaning
μ	: Birthrate in humans is assumed same with death rate
μ_b	: Birthrate in birds is assumed same with death rate
β	: The probability of infectious contact was happen in humans
β_b	: The probability of infectious contact was happen in birds
m	: Death rate of infected human (assumed equal to μ)
m_b	: Death rate of infected bird (assumed equal to μ_b)
Υ	: Recovery rate of infected human
θ	: Immunity loss rate
δ	: The proportion of susceptible bird to be vaccinated

where N is the total human population, S is total number of susceptible person, I is total number of infected person, R is total number of recovered person, N_b is the total bird population, S_b is total number of susceptible bird, I_b is total number of infected bird, and v_b is total number of vaccinated bird. The meaning of parameter in model were given in Table 1.

From Fig. 1 we construct te system of ordinary differential equation as System (1).

$$
\begin{aligned}
\frac{dS}{dt} &= \mu N + \theta R - S\left(\beta \frac{I_b + I}{N} + \mu\right) \\
\frac{dI}{dt} &= \beta \frac{S}{N}(I_b + I) - (m + y)I \\
\frac{dR}{dt} &= yI - (\theta + \mu)R \\
\frac{dS_b}{dt} &= \mu_b N_b - \left(\beta_b \frac{I_b}{N_b} + \delta + \mu_b\right)S_b \\
\frac{dI_b}{dt} &= \beta_b \frac{S_b}{N_b} I_b - m_b I_b \\
\frac{dV_b}{dt} &= \delta S_b - \mu_b v_b \\
S + I + R &= N \\
S_b + I_b + v_b &= N_b
\end{aligned} \tag{1}
$$

We assumed that $m = \mu$ and $m_b = \mu_b$ then we get System (2).

$$
\begin{aligned}
\frac{dS}{dt} &= \mu N + \theta R - S\left(\beta \frac{I_b + I}{N} + \mu\right) \\
\frac{dI}{dt} &= \beta \frac{S}{N}(I_b + I) - (\mu + y)I \\
\frac{dR}{dt} &= yI - (\theta + \mu)R \\
\frac{dS_b}{dt} &= \mu_b N_b - \left(\beta_b \frac{I_b}{N_b} + \delta + \mu_b\right)S_b \\
\frac{dI_b}{dt} &= \beta_b \frac{S_b}{N_b} I_b - \mu_b I_b \\
\frac{dV_b}{dt} &= \delta S_b - \mu_b V_b \\
S + I + R &= N \\
S_b + I_b + v_b &= N_b
\end{aligned} \tag{2}
$$

Clear that $\frac{dN}{dt} = 0 \leftrightharpoons N = K > 0, k \in R$ and $\frac{dN_b}{dt} = 0 \leftrightharpoons N = L > 0, L \in R$.

Hence, we get System (3)

$$
\begin{aligned}
\frac{dS}{dt} &= (\mu + \theta)K - \theta I - S\left(\beta \frac{I_b + I}{K} + \mu + \theta\right) \\
\frac{dI}{dt} &= \beta \frac{S}{K}(I_b + I) - (\mu + y)I \\
\frac{dS_b}{dt} &= \mu_b L - \left(\beta_b \frac{I_b}{L} + \delta + \mu_b\right)S_b \\
\frac{dI_b}{dt} &= \beta_b \frac{S_b}{L} I_b - \mu_b I_b
\end{aligned} \tag{3}
$$

Domain of System (3) is defined

$$\Gamma = \{P \in R^4 \mid P = (S, I, S_b, I_b) \text{ where } 0 \leq S, I < K \text{ and } 0 \leq S_b, I_b < L\}$$

The existence of equilibrium points of System (3) is given in Theorem 1.

Theorem 1.

Let $r_0 = \dfrac{\beta_b}{\mu_b + \delta}$ and $R_0 = \dfrac{\beta}{\mu + \gamma}$.

If $r_0 < 1$ and $R_0 < 1$ then System (3) has only one equilibrium point i.e. non endemic equilibrium point $P_0 = (s, I, s_b, I_b) = P = (K, 0, \frac{\mu_b L}{\delta + \mu_b}, 0)$.

1. If $r_0 < 1$ and $R_0 > 11$ then System (3) has two equilibrium i.e P_0 and
$$P_1 = (s, I, s_b, I_b) = \left(\frac{K(\mu+\gamma)}{\beta}, \frac{K(\mu+\theta)[\beta-(\mu+\gamma)]}{\beta(\mu+\gamma+\theta)}, \frac{\mu_b L}{\delta+\mu_b}, 0\right)$$
2. if $r_0 > 1$ and $R_0 > 1$ then System (3) has Three equilibrium i.e P_0, P_1, and
$$P_2 = (s, I, s_b, I_b) = \left(\frac{K(\mu+\gamma)I^*}{\beta(I_b^* + I^*)}, I^*, \frac{\mu_b . L}{\beta_b}, I_b^*\right)$$
where $I_b^* = \frac{L[\beta_b - (\mu_b+\delta)]}{\beta_b}$, $I^* = \frac{-B+\sqrt{B^2-4AC}}{2A}$, $A = \beta(\theta + \mu + y)$,
$B = \beta(\theta + \mu + y)I_b^* - (\mu + \theta)K[\beta - (\mu + y)]$, and $C = -\beta(\mu + \theta)KI_b^*$.

Proof:

The equilibrium points were solutions of System (4).

$$(\mu+\theta)K - \theta I^* - S^*\left(\beta\frac{I_b^* + I^*}{K} + \mu + \theta\right) = 0$$
$$\beta\frac{S^*}{K}(I_b^* + I^*) - (\mu + y)I^* = 0$$
$$\mu_b L - (\beta_b \frac{I_b^*}{L} + \delta + \mu_b)s_b^* = 0 \qquad (4)$$
$$\beta_b\frac{S_b^*}{L}I_b^* - \mu_b I_b^* = 0$$

From the fourth equation of System (4), we get

$$\left[\beta_b\frac{S_b^*}{L} - \mu_b\right]I_b = 0 \Leftrightarrow I_b^* = 0 \vee \beta_b\frac{S_b^*}{L} - \mu_b = 0 \Leftrightarrow I_b = 0 \vee S_b^* = \frac{\mu_b . L}{\beta_b}.$$

The case of $I_b^* = 0$:

Substitute the value of I_b^* to the third equation, we get $s_b^* = \frac{\mu_b L}{\delta + \mu_b}$.

Substitute the value of I_b^* to the second equation, we get $I^* = 0$ v $s^* = \frac{K(\mu+\gamma)}{\beta}$.

The case of $I^* = 0$:

For this case, we get $P_0 = (s, I, s_b, I_b) = (K, 0, \frac{\mu_b L}{\delta+\mu_b}, 0)$.

The case of $I^* \neq 0$:

Clear that $s^* = \frac{K(\mu+\gamma)}{\beta}$. Substitute to the first equation then we get $I^* = \frac{K(\mu+\theta)[\beta-(\mu+\gamma)]}{\beta(\mu+\gamma+\theta)}$.

Clear that if $R_0 = \frac{\beta}{\mu+\gamma} > 1$ then $I^* > 0$. Hence, we get if $R_0 > 1$ and $\frac{\theta}{\mu+\gamma} < 1$ then

$$P_1 = (s, I, s_b, I_b) = \left(\frac{K(\mu+\gamma)}{\beta}, \frac{K(\mu+\theta)[\beta-(\mu+\gamma)]}{\beta(\mu+\gamma+\theta)}, \frac{\mu_b L}{\delta+\mu_b}, 0\right)$$

The case of $I_b^* \neq 0$:

Clear that $S_b^* = \frac{\mu_b . L}{\beta_b}$. Substitute the value of S_b^* to the third equation, we get $I_b^* = \frac{L[\beta_b - (\mu_b+\delta)]}{\beta_b}$.

Clear that $I_b^* > 0$ if $r_0 = \frac{\beta_b}{\mu_b+\delta} > 1$. From the second equation, we get $S^* = \frac{K(\mu+\gamma)I^*}{\beta(I_b^*+I^*)}$.

Substitute to the first equation, then we get

$\beta(\theta + \mu + \gamma)I^{*2} + [\beta(\theta + \mu + \gamma)I_b^* - (\mu + \theta)K[\beta - (\mu + \gamma)]]I^* - \beta(\mu + \theta)KI_b^* = 0.$

Let $A = \beta(\theta + \mu + \gamma)$, $B = \beta(\theta + \mu + \gamma)I_b^* - (\mu + \theta)K[\beta - (\mu + \gamma)]$, and $C = -\beta(\mu + \theta)KI_b^*$.

Clear that $A > 0$ and $C < 0$. Hence $B^2 - 4AC > B^2 > 0$.

Hence $I_1^* = \frac{-B-\sqrt{B^2-4AC}}{2A} < 0$ and $I_2^* = \frac{-B+\sqrt{B^2-4AC}}{2A} > 0$.

Hence, $I^* = I_2^* = \frac{-B+\sqrt{B^2-4AC}}{2A}$ as the positive root of the equation if $R_0 > 1$.

Hence, $S^* = \frac{K(\mu+\gamma)I^*}{\beta(I_b^*+I^*)} > 0$. Hence, if $r_0 > 1, R_0 > 1$ and $\frac{\theta}{\mu+\gamma} > 1$ then there are exists

$$P_2 = (S, I, S_b, I_b) = \left(\frac{K(\mu+\gamma)I^*}{\beta(I_b^*+I^*)}, I^*, \frac{\mu_b . L}{\beta_b}, I_b^*\right)$$

where $I_b^* = \frac{L[\beta_b - (\mu_b+\delta)]}{\beta_b}$, $I^* = \frac{-B+\sqrt{B^2-4AC}}{2A}$, $A = \beta(\theta + \mu + \gamma)$, $B = \beta(\theta + \mu + \gamma)I_b^* - (\mu + \theta)K[\beta - (\mu + \gamma)]$, and $C = -\beta(\mu + \theta)KI_b^*$.

The Stability of equilibrium points of System (3) is given in Theorem 2.

Theorem 2.
Let $r_0 = \frac{\beta_b}{\mu_b+\delta}$ and $R_0 = \frac{\beta}{\mu+\gamma}$.

1. If $r_0 < 1$ and $R_0 < 1$ then P_0 is locally asymptotically stable
2. If $r_0 < 1$ and $R_0 > 1$ then P_0 is unstable and P_1 is locally asymptotically stable.
3. if $r_0 > 1$ and $R_0 > 1$ then P_0 and P_1 are unstable, and P_2 is locally asymptotically stable.

Proof:

The Jacobian matrix of System (4) was given below

$$J_b(P) = \begin{bmatrix} -\frac{\beta(I_b+I_h)}{K} - (\mu+\theta) & -\theta - \frac{\beta S}{K} & 0 & -\frac{\beta S}{K} \\ \frac{\beta(I_b+I_h)}{K} & \frac{\beta S}{K} - (\mu+\gamma) & 0 & \frac{\beta S}{K} \\ 0 & 0 & -\frac{\beta_b I_b}{L} - (\delta+\mu_b) & -\frac{\beta_b S_b}{L} \\ 0 & 0 & \frac{\beta_b I_b}{L} & \frac{\beta_b S_b}{L} - \mu_b \end{bmatrix}$$

For $P_0 = (S, I, S_b, I_b) = (K, 0, \frac{\mu_b L}{\delta+\mu_b}, 0)$:

The eigen values of $J_b(P_0)$ are $\lambda_1 = -(\mu + \theta)$, $\lambda_2 = \beta - (\mu + \gamma)$, $\lambda_3 = -(\delta + \mu_b)$, and $\lambda_4 = \frac{\mu_b[\beta_b - (\delta + mu_b)]}{\delta + \mu_b}$.

Clear that λ_1 and λ_3 are negative, $\lambda_2 < 0$ if $R_0 < 1$, and $\lambda_4 < 0$ if $r_0 < 1$. Hence, (1) P_0 is locally asymptotically stable if $R_0 < 1$ and $r_0 < 1$ and (2) P_0 is unstable if $R_0 > 1$.

For $P_1 = (s, I, s_b, I_b) = \left(\frac{K(\mu+\gamma)}{\beta}, \frac{K(\mu+\theta)[\beta-(\mu+\gamma)]}{\beta(\mu+\gamma+\theta)}, \frac{\mu_b L}{\delta+\mu_b}, 0\right)$:

This analysis was only done at $R_0 > 1$. The characteristics polynomial of $J_b(P_1)$ is

$$\frac{1}{(\mu+\gamma+\theta)(\delta+\mu_b)}\{(\lambda+\delta+\mu_b)[(\delta+\mu_b)\lambda+\mu_b(\delta+\mu_b-\beta_b)][(\mu+y+\theta)\lambda^2 + (\beta+\theta)(\mu+\theta)\lambda+(\mu+\theta)(\mu+y-\theta)(\beta-\mu-y)]\} = 0$$

From the characteristics polynomial of $J_b(P_1)$, we got two first eigenvalues i.e $\lambda_1 = -(\delta+\mu_b)$ and $\lambda_2 = \frac{\mu_b(\beta_b-\delta-\mu_b)}{\delta+\mu_b}$. Clear that $\lambda_1 < 0$ and $\lambda_2 < 0$ if $r_0 < 1$.

From $(\mu+y+\theta)\lambda^2 + (\beta+\theta)(\mu+\theta)\lambda + (\mu+\theta)(\mu+y-\theta)(\beta-\mu-y) = 0$, we got the simpler equation $\lambda^2 + \frac{(\beta+\theta)(\mu+\theta)}{(\mu+\gamma+\theta)}\lambda + (\mu+\theta)(\beta-\mu-y) = 0$.

Define $A = 1$, $B = \frac{(\beta+\theta)(\mu+\theta)}{(\mu+y+\theta)}$, and $C = (\mu+\theta)(\beta-\mu-y)$.

Clear that $B > 0$ and $C > 0$ if $R_0 > 1$. Hence, $\lambda_3 = \frac{-B-\sqrt{D}}{2}$ and $\lambda_4 = \frac{-B+\sqrt{D}}{2}$.

where $D = B^2 - 4C$. Because $C > 0$ then $D < B^2$. Hence, $Re(\lambda_1) < 0$ and $Re(\lambda_2) < 0$ if $R_0 > 1$. Hence, (1) P_1 is locally asymptotically stable if $r_0 < 1$ and $R_0 > 1$; (2) P_1 is unstable if $r_0 > 1$.

For $P_2 = (s, I, s_b, I_b) = \left(\frac{K(\mu+\gamma)I^*}{\beta(I_b^*+I^*)}, I^*, \frac{\mu_b . L}{\beta_b}, I_b^*\right)$:

We have $I_b^* = \frac{L[\beta_b-(\mu_b+\delta)]}{\beta_b}$, $I^* = \frac{-B+\sqrt{B^2-4AC}}{2A}$, where $A = \beta(\theta+\mu+y)$, $B = \beta(\theta+\mu+y)I_b^* - (\mu+\theta)K[\beta-(\mu+y)]$, and $C = -\beta(\mu+\theta)KI_b^*$.

This analysis was only done at $r_0 > 1$ and $R_0 > 1$. The characteristics polynomial of $J_b(P_2)$ is

$$\frac{1}{K(I_b^*+I_h^*)L}\{[L\lambda^2 + (L(\delta+\mu_b)+\beta_b I_b^*)\lambda + \mu_b\beta_b I_b^*][A_1\lambda^2 + B_1\lambda + C_1]\} = 0$$

where $A_1 = K(I_b^*+I_h^*)$, $B_1 = \beta I_h^{*2} + (2\beta I_b^* + K(\mu+\theta))I_h^* + I_b^*[\beta I_b^* + K(2\mu+\theta+y)]$,

$C_1 = \beta(\mu+y+\theta)I_h^{*2} + 2\beta I_b^*(\mu+y+\theta)I_h^* + I_b^*[\beta I_b^*(\mu+y+\theta) + K(\mu+\theta)(\mu+y)]$.

From $L\lambda^2 + [L(\delta+\mu_b)+\beta_b I_b^*]\lambda + \mu_b\beta_b I_b = 0$ we got

$$\lambda_1 = \frac{-(L(\delta+\mu_b)+\beta_b I_b^*) - \sqrt{[L(\delta+\mu_b)+\beta_b I_b^*]^2 - 4L.\mu_b\beta_b I_b^*}}{2L} \text{ and}$$

$$\lambda_2 = \frac{-(L(\delta+\mu_b)+\beta_b I_b^*) + \sqrt{[L(\delta+\mu_b)+\beta_b I_b^*]^2 - 4L.\mu_b\beta_b I_b^*}}{2L}.$$

Clear that $[L(\delta+\mu_b)+\beta_b I_b^*]^2 - 4L.\mu_b\beta_b I_b^* = [(\beta_b I_b^* - L\mu_b)^2 + 2L\delta\beta_b I_b^*] > 0$ and $[L(\delta+\mu_b)+\beta_b I_b^*]^2 > [L(\delta+\mu_b)+\beta_b I_b^*]^2 - 4L.\mu_b\beta_b I_b^* > 0$.

Hence, λ_1 and λ_2 are negative. From $A_1\lambda^2 + B_1\lambda + C_1 = 0$ where $A_1 = K(I_b^*+I_h^*)$,

$B_1 = \beta I_h^{*2} + (2\beta I_b^* + K(\mu+\theta))I_h^* + I_b^*[\beta I_b^* + K(2\mu+\theta+y)]$,

$C_1 = \beta(\mu+y+\theta)I_h^{*2} + 2\beta I_b^*(\mu+y+\theta)I_h^* + I_b^*[\beta I_b^*(\mu+y+\theta) + K(\mu+\theta)(\mu+y)]$

we got $\lambda_3 = \frac{-B_1 - \sqrt{B_1^2 - 4A_1C_1}}{2A}$ and $\lambda_3 = \frac{-B+\sqrt{B^2-4AC}}{2A}$. Clear that A, B, and C are all positive if $r_0 > 1$ and $R_0 > 1$. Hence, $B_1^2 - 4A_1C_1 < B_1^2$. It caused λ_3 and λ_4 have real part.

Hence, P_2 is locally asymptotically stable if $r_0 > 1$ and $R_0 > 1$.

4. Simulation

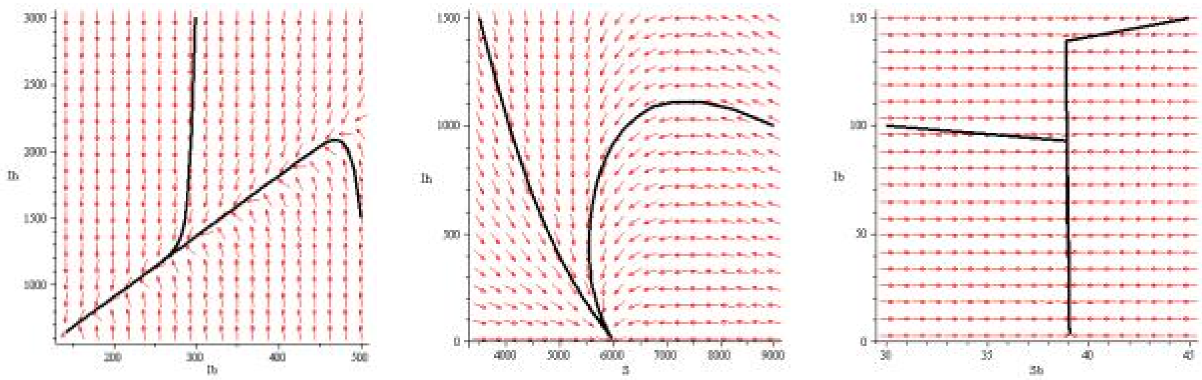
Simulation was done for three cases like three conditions in Theorem 2. Value of some parameter followed from Kharis & Arifudin [12]. Value of parameter were given in Table 2.

Table 2. Value of parameters

Parameter	Value	Parameter	Value
μ	0,00004	β_b	0 to 1
β	0 to 1	m_b	0,00137
m	0,00004	δ	0,7
y	0,098	K	6000
θ	0,037	L	20000
μ_b	0,00137		

4.1. Simulation for $r_0 < 1$ and $R_0 < 1$

In this case, we used the value $\beta = 0,08$ and $\beta_b = 0,5$. From the formula r_0 and R_0 in Theorem 1, we got $r_0 = 0,713 < 1$ and $R_0 = 0,816 < 1$. From Theorem 1, There is only one equilibrium point i.e. $P_0 = (s, I, s_b, I_b) = (6000,0,39.06,0)$. The graphs for this simulation were given on Figure 2.



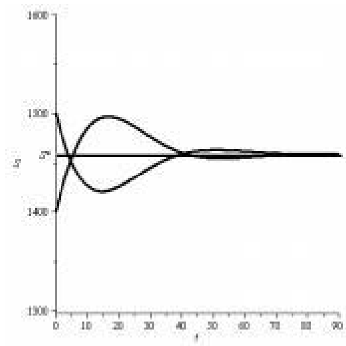
(a) Plane I_b vs I_h (b) Plane s vs I_h (c) Plane s_b vs I_b

Figure 2. Vector Field in neighborhood of point P_0 at $r_0 < 1$ and $R_0 < 1$

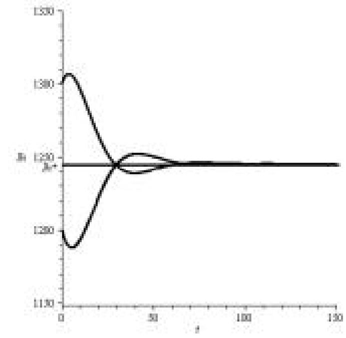
From Figure 2, it can be seen that the solutions that is near from P_0 converge to P_0. These simulations were similar with Theorem 2.

4.2. Simulation for $r_0 < 1$ and $R_0 > 1$.

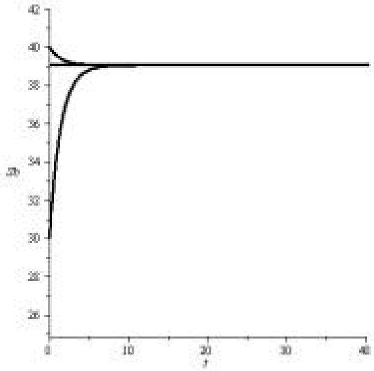
In this case, we used the value $\beta = 0,4$ and $\beta_b = 0,5$. From the formula r_0 and R_0 in Theorem 1, we got $r_0 = 0.71 < 1$ and $R_0 = 4,08 > 1$. From Theorem 1, There are two equilibrium points i.e. $P_0 = (s, I, s_b, I_b) = (6000,0,39.06,0)$ and $P_1 = (s, I, s_b, I_b) = (1470.6,1242.36,39.06,0)$. The graphs for this simulation were given on Figure 3. From Figure 3, it can be seen that the solutions that is near from P_1 converge to P_1. These simulations were similar with Theorem 2.



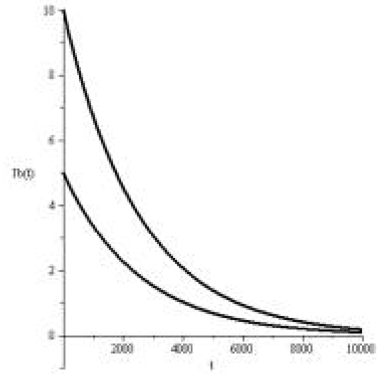
(a) $S(t)$



(b) $I(t)$

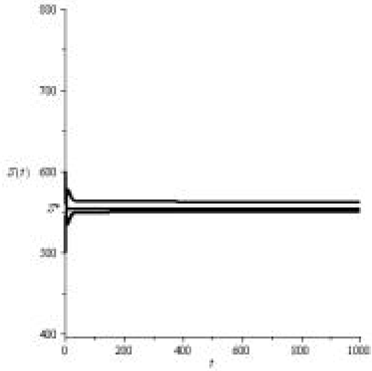


(c) $S_b(t)$

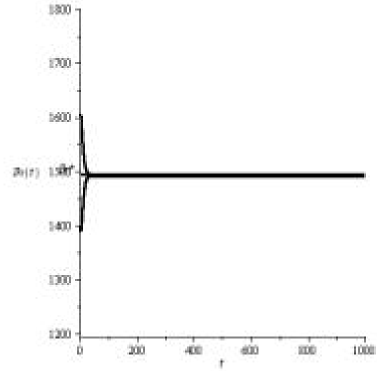


(d) $I_b(t)$

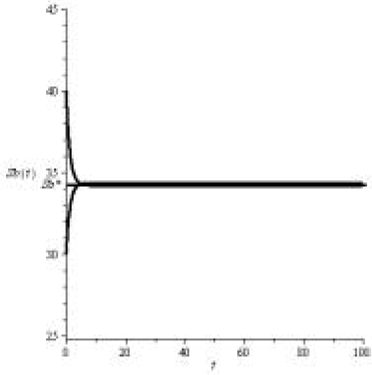
Figure 3. Phase portrait projection of solution at $r_0 < 1$ and $R_0 > 1$



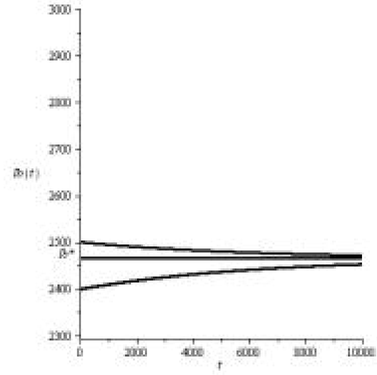
(a) $S(t)$



(b) $I(t)$



(c) $S_b(t)$



(d) $I_b(t)$

Figure 4. Phase portrait projection of solution at $r_0 > 1$ and $R_0 > 1$

4.3. Simulation for $r_0 > 1$ and $R_0 > 1$

In this case, we used the value $\beta = 0{,}4$ and $\beta_b = 0{,}8$. From the formula r_0 and R_0 in Theorem 1, we got $r_0 = 1{,}14 > 1$, $R_0 = 4{,}08 > 1$. From Theorem 1, There are three equilibrium points i.e. $P_0 = (s, I, s_b, I_b) = (6000,0,39.06,0)$, $P_1 = (s, I, s_b, I_b) = (1470.6,1242.36,39.06,0)$, and $P_2 = (s, I, s_b, I_b) = (554.75, 1493.57, 34.25,2465.75)$. The graphs for this simulation were given on Figure 4. From Figure 4, it can be seen that the solutions that is near from P_2 converge to P_2. These simulations were similar with Theorem 2.

5. Conclusion

From analysis above, we get the dynamic of mathematics model of AI-epidemic with vaccination on bird population especially for constant population. We also got the formula of reproduction number (r_0 and R_0) which can be used to determine whether the epidemic spread widely or vanish. For the formula of r_0, we got that the proportion of vaccinated susceptible bird can change the value of r_0. If this proportion increase then r_0 decrease. It means we can prevent the spreading of this epidemic in bird population by increasing the proportion of vaccinated susceptible bird. For human population, we can prevent the spreading of this epidemic by reducing the probability of infectious contact between infected people and susceptible people. It can be done by quarantine infected people. For the next research, we propose to make the mathematics model for non-constant population.

Acknowledgments

This research was funded by Ministry of Technology Research and Higher Education (Kemristekdikti) especially in scheme of Penelitian Produk Terapan in 2017.

References

[1] Perdue ML 2008 *Molecular Determinants of Pathogenicity for Avian Influenza Viruses*. *Avian Influenza* ed D E. Swayne (Blackwell Publishing)

[2] Sedyaningsih ER, Setiawaty V, Rif'ati L, Harun S, Heriyanto B, Nur APK, Apsari PH, Isfarandi S, Sarivadi E, Saptiawati C and Tresnaningsih E 2006 Bul Penel Kesehatan **34** 137

[3] Scoones I and Forster P 2008 *The International Response to Highly Pathogenic Avian Influenza: Science, Policy and Politics*, STEPS Working Paper 10 (Brighton: STEPS Centre)

[4] Yang Y, Sugimoto J D, Halloran M E, Basta N E, Chao D L, Matrajt L, Potter G, Kenah E and Longini Jr. IM 2009 Science **326** 729

[5] Jansen A G S C, Sanders E A M , Hoes A W, van Loon A M and Hak E 2007 Eur Respir J **30** 1158

[6] Widiasih D A, SusetyaH, Sumiarto B, Tabbu C R and Budiharta S 2006 J Sain Vet **24** 71

[7] de Jong M D, Thanh T T, Khanh T H, Hien V M, Smith G J D, Chau N V, Cam B V, Qui P T, Ha D Q, GuanY, Peiris J S M, Hien T T and Farrar J 2005 *N Engl J Med* **353** 2667

[8] Gooskens J, Jonges M, Claas ECJ, Meijer A, van den Broek PJ, and Kroes ACM 2009 JAMA **301** 1042

[9] TuncerN and Martcheva M 2013 J Biol Syst **21** 1340004

[10] Bourouiba L 2013 *CAB Reviews* **17**

[11] Vemula SV, Zhao J, Liu J, Wang X, Biswas S and Hewlett I 2016 Viruses **8** 96

[12] Kharis M and Arifudin R 2017 Int J Pure Appl Math **112** 571